

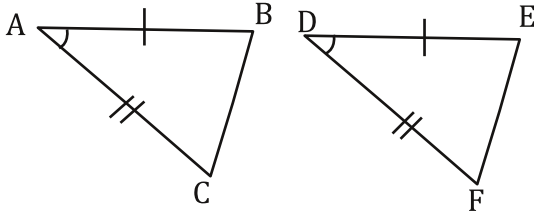
Proofs Involving Congruent Triangles

First, let's analyze some proofs.

This is easy! All you have to do is explain in plain English what is going on in the proofs. We'll look at some examples first.

AE. 1.

Given: $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $\angle A \cong \angle D$

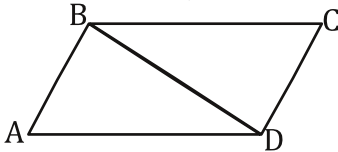


Prove: $\triangle ABC \cong \triangle DEF$

Statements	Reasons
1. $\overline{AB} \cong \overline{DE}$	1. Given
2. $\overline{AC} \cong \overline{DF}$	2. Given
3. $\angle A \cong \angle D$	3. Given
4. $\triangle ABC \cong \triangle DEF$	4. SAS

AE. 2.

Given: $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$

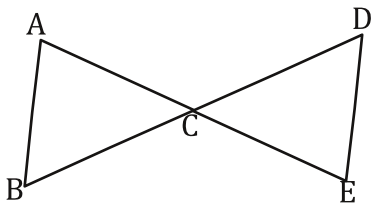


Prove: $\triangle ABD \cong \triangle CBD$

Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$	1. Given
2. $\overline{AD} \cong \overline{CB}$	2. Given
3. $\overline{BD} \cong \overline{BD}$	3. Reflexive property
4. $\triangle ABD \cong \triangle CBD$	4. SSS

AE. 3.

Given: \overline{AE} Bisects \overline{BD} , $\angle B \cong \angle D$



Prove: $\triangle ABC \cong \triangle DBC$

Statements	Reasons
1. $\angle B \cong \angle D$	1. Given
2. \overline{AC} Bisects \overline{BD}	2. Given
3. $\overline{BC} \cong \overline{DC}$	3. Definition of Bisect
4. $\angle ACB \cong \angle DCE$	4. Vertical angles
5. $\triangle ABC \cong \triangle DBC$	5. ASA

Analysis:

Working backward we must ask the key question, "How can we show that two triangles are congruent?" The answer? A triangle congruence theorem like SSS, SAS, ASA, AAS or HL. This gives us B1: $\triangle ABC \cong \triangle DEF$, by some property, but which one? To find out, start working forward. Listing all of the given information gives us a pair of angles $\angle A$ and $\angle D$ sandwiched between a pair of congruent sides $\overline{AB} \cong \overline{DE}$ and $\overline{AC} \cong \overline{DF}$. So this means we have $\triangle ABC \cong \triangle DEF$ by the SAS theorem which is B2: and the proof is complete.

Analysis:

Working backward, we must ask the key question "How can we show that two triangles are congruent?" The answer? A triangle congruence theorem like SSS, SAS, ASA, AAS or HL. This gives us B1: $\triangle ABC \cong \triangle BCD$ by some property, but which one? Then start working forward. Listing all of the given information gives us two pairs of sides $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$, but this is not enough. We need another pair of sides or an angle between them. Looking now at the diagram we have $\overline{BD} \cong \overline{BD}$ as a shared line. So this brings us to say $\triangle ABC \cong \triangle BCD$ by SSS which is B1 and the proof is complete.

Analysis:

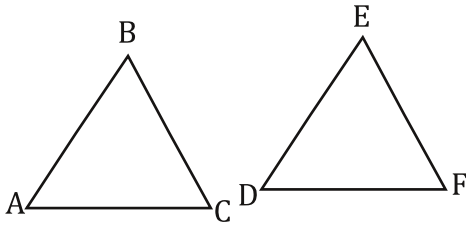
Working backward we must ask the key question, "How can we show that two triangles are congruent?" The answer? A triangle congruence theorem like SSS, SAS, ASA, AAS or HL. This gives us B1: $\triangle ABC \cong \triangle BCD$ by some property, but which one? Then start working forward. Listing all of the given information gives us a pair of angles $\angle B$ and $\angle D$, and \overline{BD} and \overline{AE} bisects \overline{BD} . If \overline{AE} bisects \overline{BD} then $\overline{BC} \cong \overline{DC}$! This is not enough though. Looking at the diagram we see vertical angles $\angle ACB \cong \angle DCE$, which gives us $\triangle ABC \cong \triangle BCD$ by the property ASA. This is B1 and the proof is complete.

Your turn! Write an analysis of each proof involving congruent triangles.

1.

Analysis:

Given: $\overline{BC} \cong \overline{EF}$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$



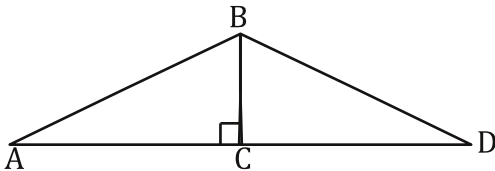
Prove: $\triangle ABC \cong \triangle DEF$

Statements	Reasons
1. $\overline{BC} \cong \overline{EF}$	1. Given
2. $\angle B \cong \angle E$	2. Given
3. $\angle C \cong \angle F$	3. Given
4. $\triangle ABC \cong \triangle DEF$	4. ASA

2.

Analysis:

Given: $\overline{AB} \cong \overline{BD}$



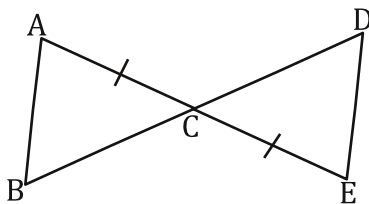
Prove: $\triangle ABC \cong \triangle DBC$

Statements	Reasons
1. $\overline{AB} \cong \overline{BD}$	1. Given
2. $\overline{BC} \cong \overline{BC}$	2. Reflexive property
3. $\triangle ABC \cong \triangle DBC$	3. HL

3.

Analysis:

Given $\overline{AB} \parallel \overline{ED}$, $\overline{AC} \cong \overline{EC}$



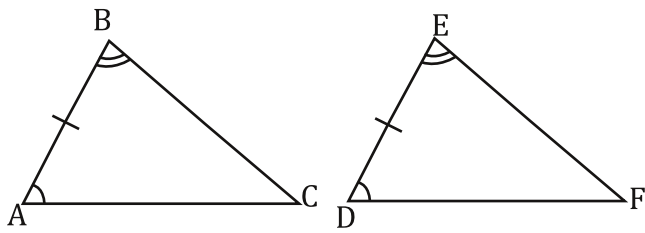
Prove: $\triangle ABC \cong \triangle EDC$

Statements	Reasons
1. $\overline{AB} \parallel \overline{ED}$	1. Given
2. $\overline{AC} \cong \overline{EC}$	2. Given
3. $\angle A \cong \angle E$	3. Alternate Interior angles
4. $\angle ACB \cong \angle DCE$	4. Vertical angles
5. $\triangle ABC \cong \triangle EDC$	5. ASA

For these fill in any missing statements or reasons.

1.

Given: $\overline{AB} \cong \overline{DE}$, $\angle B \cong \angle E$, and $\angle A \cong \angle D$

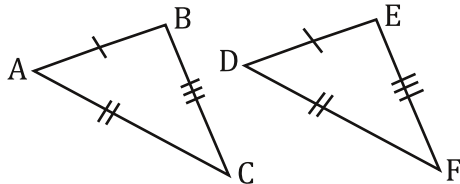


Prove: $\triangle ABC \cong \triangle DEF$

Statements	Reasons
1. $\overline{AB} \cong \overline{DE}$	1. Given
2.	2. Given
3. $\angle A \cong \angle D$	3.
4. $\triangle ABC \cong \triangle DEF$	4.

3.

Given: $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $\overline{BC} \cong \overline{EF}$

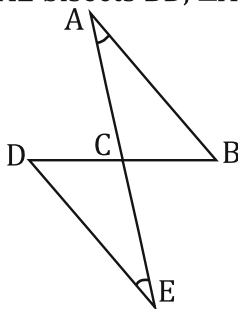


Prove: $\triangle ABC \cong \triangle DEF$

Statements	Reasons
1. $\overline{AB} \cong \overline{DE}$	1.
2.	2.
3.	3.
4.	4. SSS

5.

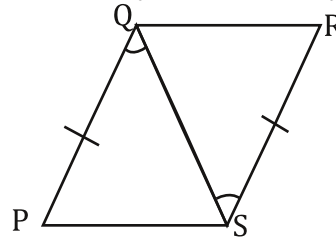
Given: \overline{AE} bisects \overline{BD} , $\angle A \cong \angle E$



Prove: $\triangle ABC \cong \triangle EDC$

Statements	Reasons
1. $\angle A \cong \angle E$	1.
2.	2. Given
3.	3. Definition of Bisect
4. $\angle ACB \cong \angle DCE$	4.
5. $\triangle ABC \cong \triangle EDC$	5.

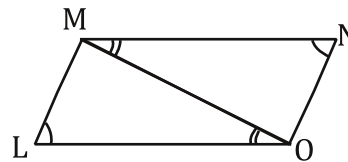
2. Given: $\overline{PQ} \cong \overline{RS}$, and $\angle PQS \cong \angle RSQ$



Prove: $\triangle PQS \cong \triangle RSQ$

Statements	Reasons
1.	1. Given
2.	2. Given
3. $\overline{QS} \cong \overline{QS}$	3.
4. $\triangle PQS \cong \triangle RSQ$	4.

4. Given: $\angle L \cong \angle N$, $\angle LOM \cong \angle NMO$

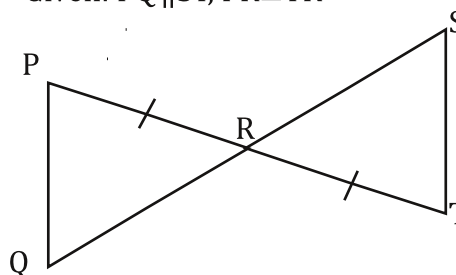


Prove: $\triangle LMO \cong \triangle NMO$

Statements	Reasons
1.	1.
2.	2. Given
3.	3. Reflexive Property
4. $\triangle LMO \cong \triangle NMO$	4.

6.

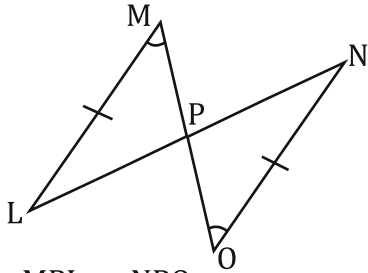
Given: $\overline{PQ} \parallel \overline{ST}$, $\overline{PR} \cong \overline{TR}$



Prove: $\triangle PQR \cong \triangle TSR$

Statements	Reasons
1. $\overline{PR} \cong \overline{TR}$	1.
2.	2. Given
3. $\angle P \cong \angle T$	3.
4. $\angle ACB \cong \angle DCE$	4.
5.	5. ASA

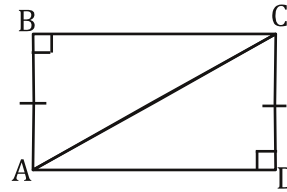
7. Given: $\overline{LM} \cong \overline{NO}$, and $\angle M \cong \angle O$



Prove: $\triangle MPL \cong \triangle NPO$

Statements	Reasons
1. $\overline{LM} \cong \overline{NO}$	1.
2.	2. Given
3.	3.
4.	4. AAS

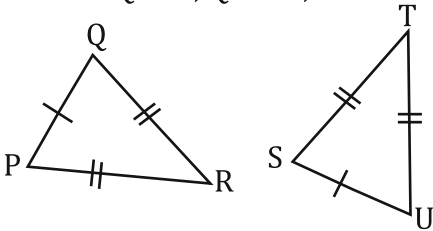
8. Given: $\overline{AB} \cong \overline{DC}$



Prove: $\triangle ABC \cong \triangle CDA$

Statements	Reasons
1.	1. Given
2. $\overline{AC} \cong \overline{AC}$	2.
3. $\triangle ABC \cong \triangle CDA$	3.

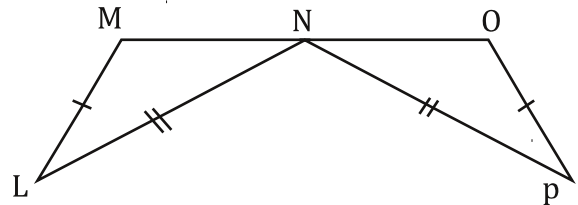
9. Given: $\overline{PQ} \cong \overline{SU}$, $\overline{QR} \cong \overline{ST}$, and $\overline{PR} \cong \overline{TU}$



Prove: $\triangle PQR \cong \triangle STU$

Statements	Reasons
1.	1. Given
2.	2. Given
3.	3.
4. $\triangle PQR \cong \triangle STU$	4.

10. Given: N is the midpoint of \overline{MO} , $\overline{LM} \cong \overline{OP}$, and $\overline{LN} \cong \overline{PN}$

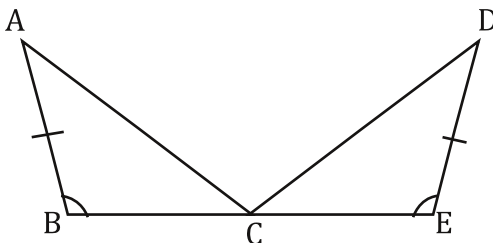


Prove: $\triangle LMN \cong \triangle PON$

Statements	Reasons
1. $\overline{LM} \cong \overline{OP}$	1. Given
2. $\overline{LN} \cong \overline{PN}$	2.
3. N is the Midpoint of \overline{MO}	3. Given
4.	4. Midpoint
5.	5. SSS

11.

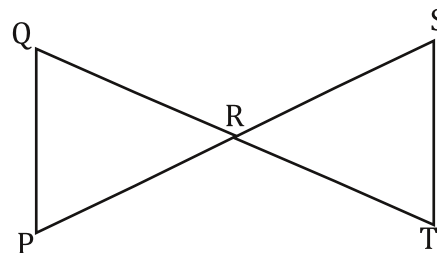
Given: C is the midpoint of \overline{BE} , $\angle B \cong \angle E$, and $\overline{AB} \cong \overline{DE}$



Prove: $\triangle ABC \cong \triangle DEC$

Statements	Reasons
1. $\angle B \cong \angle E$	1.
2. $\overline{AB} \cong \overline{DE}$	2.
3.	3. Given
4.	4. Midpoint
5. $\triangle ABC \cong \triangle DEC$	5. SAS

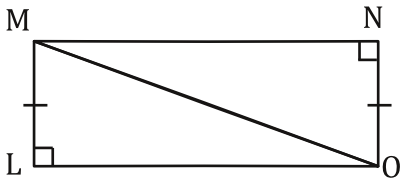
12. Given: \overline{QT} bisects \overline{SP} , \overline{SP} bisects \overline{QT}



Prove: $\triangle QRP \cong \triangle SRT$

Statements	Reasons
1. \overline{QT} bisects \overline{SP}	1. Given
2.	2. Given
3. $\overline{QR} \cong \overline{TR}$	3. Definition of Bisect
4. $\overline{PR} \cong \overline{SR}$	4.
5.	5. Vertical Angles
6. $\triangle QRP \cong \triangle SRT$	6.

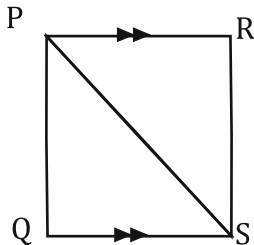
13. Given: $\overline{LM} \cong \overline{NO}$



Prove: $\triangle LMO \cong \triangle NOM$

Statements	Reasons
1. $\overline{LM} \cong \overline{NO}$	1.
2.	2.
3.	3.

15. Given: $\overline{PR} \parallel \overline{QS}$, $\angle QPS \cong \angle RSP$

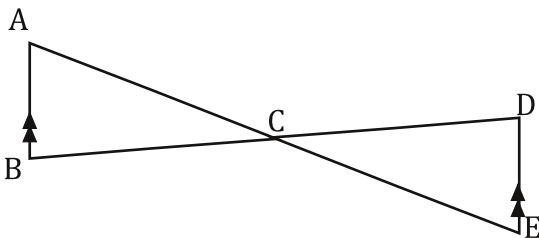


Prove: $\triangle PQS \cong \triangle SRP$

Statements	Reasons
1. $\overline{PR} \parallel \overline{QS}$	1.
2. $\angle QPS \cong \angle RSP$	2.
3. $\angle PSQ \cong \angle SPR$	3. Alternate Interior
4.	4. Reflexive Property
5. $\triangle PQS \cong \triangle SRP$	5.

17.

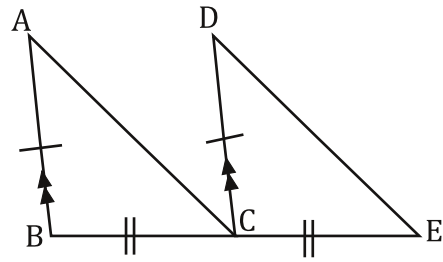
Given: \overline{AE} bisects \overline{BD} , $\overline{AB} \parallel \overline{DE}$



Prove: $\triangle ABC \cong \triangle DCE$

Statements	Reasons
1. \overline{AE} bisects \overline{BD}	1.
2.	2. Given
3. $\overline{BC} \cong \overline{DC}$	3.
4. $\angle ACB \cong \angle DCB$	4.
5.	5. Alternate Interior
6.	6. ASA

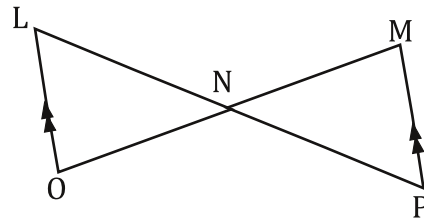
14. Given: $\overline{AB} \cong \overline{DC}$, $\overline{AB} \parallel \overline{DC}$, and $\overline{BC} \cong \overline{CE}$



Prove: $\triangle ABC \cong \triangle DCE$

Statements	Reasons
1. $\overline{AB} \cong \overline{DC}$	1. Given
2.	2. Given
3.	3. Given
4.	4. Corresponding Angles
5. $\triangle ABC \cong \triangle DCE$	5.

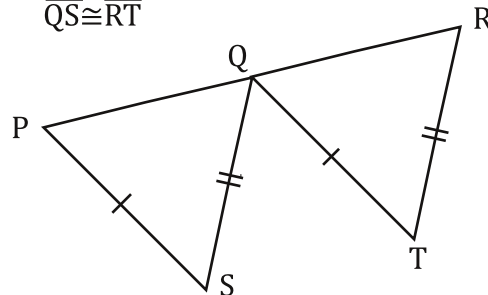
16. Given: \overline{LP} bisects \overline{MO} , $\overline{LO} \parallel \overline{MP}$



Prove: $\triangle LNO \cong \triangle MNP$

Statements	Reasons
1.	1. Given
2.	2. Given
3. $\overline{LN} \cong \overline{PN}$	3.
4.	4. Alternate Interior
5.	5. Vertical Angles
6.	6. ASA

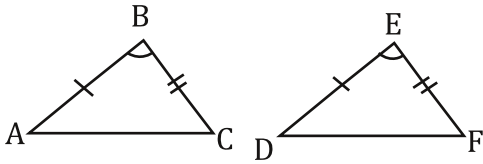
18. Given: Q is the midpoint of \overline{PR} , $\overline{PS} \cong \overline{QT}$ and $\overline{QS} \cong \overline{RT}$



Prove: $\triangle PQS \cong \triangle RQT$

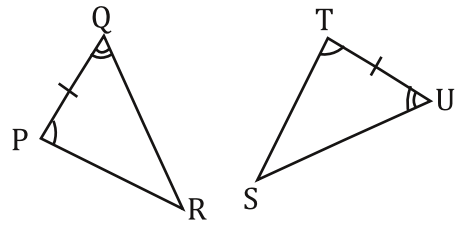
Statements	Reasons
1.	1. Given
2.	2. Given
3. $\overline{QS} \cong \overline{RT}$	3.
4.	4. Midpoint
5. $\triangle PQS \cong \triangle RQT$	5.

19. Given: $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\angle B \cong \angle E$



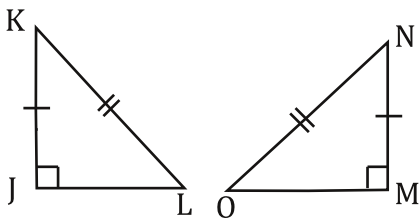
Prove: $\triangle ABC \cong \triangle DEF$

20. Given: $\overline{PQ} \cong \overline{TU}$, $\angle P \cong \angle T$, and $\angle Q \cong \angle U$



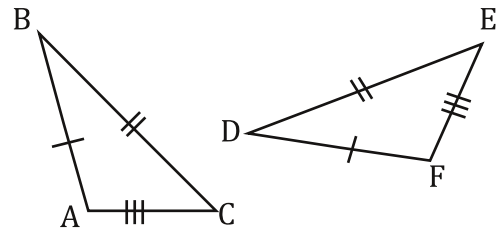
Prove: $\triangle PQR \cong \triangle TUS$

21. Given: $JK \cong MN$, $KL \cong NO$



Prove: $\triangle JKL \cong \triangle MNO$

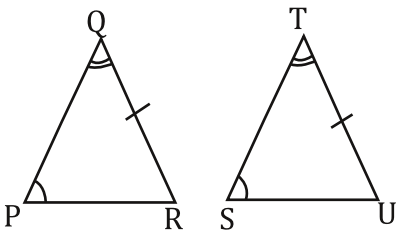
22. Given: $\overline{AB} \cong \overline{DF}$, $\overline{BC} \cong \overline{DE}$, and $\overline{AC} \cong \overline{EF}$



Prove: $\triangle ABC \cong \triangle DEF$

23.

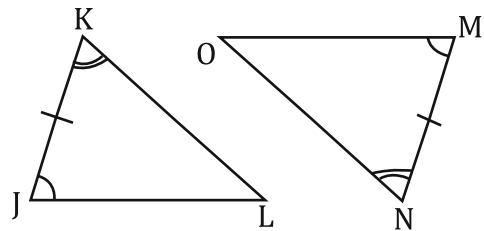
Given: $\angle P \cong \angle S$, $\angle Q \cong \angle T$, and $\overline{QR} \cong \overline{TU}$



Prove: $\triangle PQR \cong \triangle STU$

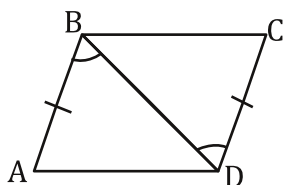
24.

Given: $\angle J \cong \angle M$, $\overline{JK} \cong \overline{MN}$ and $\angle K \cong \angle N$



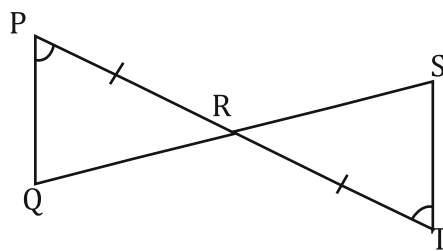
Prove: $\triangle JKL \cong \triangle MNO$

25. Given: $\overline{AB} \cong \overline{CD}$, $\angle ABD \cong \angle CDB$



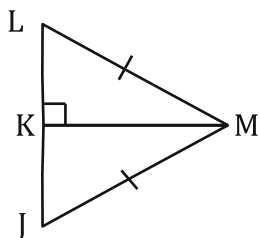
Prove: $\triangle ABD \cong \triangle CDB$

26. Given: $\overline{PR} \cong \overline{TR}$, $\angle P \cong \angle T$



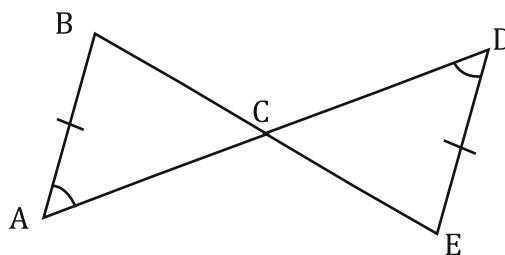
Prove: $\triangle PQR \cong \triangle TRS$

27. Given: $\overline{LM} \cong \overline{JM}$



Prove: $\triangle LKM \cong \triangle JKM$

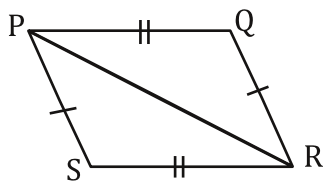
28. Given: $\overline{AB} \cong \overline{ED}$, $\angle A \cong \angle D$



Prove: $\triangle ABC \cong \triangle DCE$

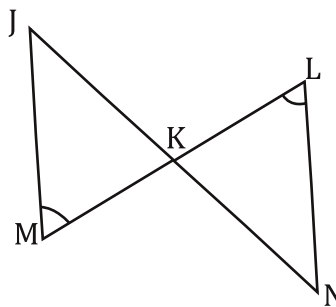
29.

Given: $\overline{PS} \cong \overline{QR}$, $\overline{PQ} \cong \overline{SR}$



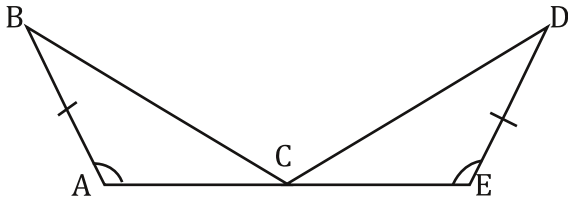
Prove: $\triangle PRS \cong \triangle RPQ$

30. Given: \overline{JN} Bisects \overline{ML} , $\angle M \cong \angle L$



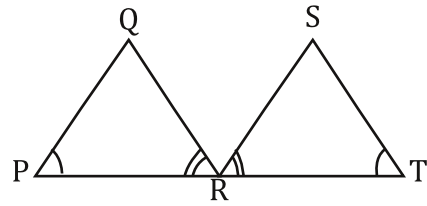
Prove: $\triangle MJK \cong \triangle LNK$

31. Given: C is the midpoint of \overline{AE} , $\overline{BA} \cong \overline{DE}$, and $\angle A \cong \angle E$



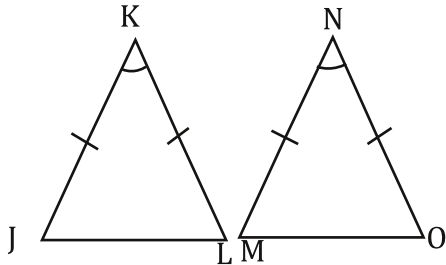
Prove: $\triangle ABC \cong \triangle EDC$

32. Given: R is the midpoint of \overline{PT} , $\angle P \cong \angle T$, and $\angle PRQ \cong \angle TRS$



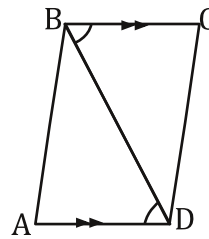
Prove: $\triangle PQR \cong \triangle TRS$

33. Given: $\angle K \cong \angle N$, $\overline{JK} \cong \overline{MN}$, $\overline{KL} \cong \overline{NO}$



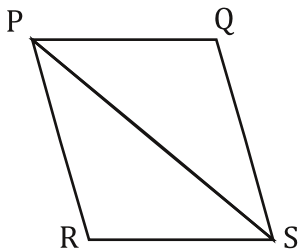
Prove: $\triangle JKL \cong \triangle MNO$

34. Given: $\overline{BA} \parallel \overline{CD}$, $\angle ADB \cong \angle CBD$



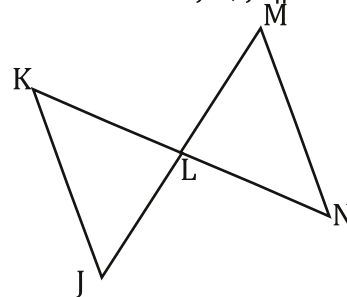
Prove: $\triangle ABD \cong \triangle CDB$

35. Given: PQRS is a parallelogram



Prove: $\triangle RPS \cong \triangle QSP$

36. Given: \overline{KN} bisects \overline{JM} , $\overline{JK} \parallel \overline{MN}$



Prove: $\triangle JKL \cong \triangle MNL$